Kantian equilibrium

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Lecture 3, INET mini-school on Inequality

The autarkic focus in economics

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3. A normative focus on self-regarding individuals

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- Henrich & Henrich (2007), Why humans cooperate. . . : anthropology

The limits of an autarkic focus: social ethos

G.A. Cohen (2009) *Why not socialism?* offers a definition of 'socialism' as a society in which earnings of individuals at first accord with equality of opportunity (Rawls 1971; Dworkin 1981; Arneson 1989; Cohen 1989), but in which inequality in those earnings is then reduced because of the necessity to maintain 'community,' an ethos in which '. . . people care about, and where necessary, care for one another, and, too, care that they care about one another.'

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But he raises a question:

. . . the principal problem that faces the socialist ideal is that we do not know how to design the machinery that would make it run. Our problem is not, primarily, human selfishness, but our lack of a suitable organizational technology: our problem is a problem of design. It may be an insoluble design problem, and it is a design problem that is undoubtedly exacerbated by our selfish propensities, but a design problem, so I think, is what we've got.

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But markets are (surely) necessary in any complex economy. How, then, can a society with social ethos achieve P-efficiency?

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The lake owned in common by a group of fishers, who each possess preferences over fish and leisure, and perhaps differential skill (or sizes of boats) in (or for) fishing. The lake produces fish with decreasing returns with respect to the fishing labour expended upon it. In the game in which each fisher proposes as her strategy a fishing time, the Nash equilibrium is inefficient due to congestion externalities.

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Ostrom: many or most of the societies in this situation learn to regulate ‘fishing,’ without privatising the ‘lake.’ Somehow, the inefficient Nash equilibrium is avoided. This example is not one in which fishers care about other fishers (necessarily), but it is one in which cooperation is organised to deal with a negative externality of autarkic behaviour.

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A vector of strategies is $\mathbf{L} = (L^1, ..., L^n) \in S^n$ and for any vector $\mathbf{L} \in S^n$, let the vector $\mathbf{L}^{-i} \in S^{n-1}$ denote the vector $\mathbf{L}$ without its ith component, $\mathbf{L}^{-i} = (L^1, ..., L^{i-1}, ..., L^{i+1}, ..., L^n)$.

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The pay-off function of player i is $V^i : S^n \to \mathbb{R}$ and the game is $G = (S, V^1, ..., V^n)$.

Kantian equilibria

A vector of strategies $\mathbf{L} = (L^1, ..., L^n) \in S^n$ is a (multiplicative) *Kantian equilibrium* of the game $G = (S, V^1, ..., V^n)$ if for all agents $i = 1, ..., n$

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Kant's categorical imperative: one should take those actions and only those actions that one would advocate all others take as well. Thus, one should expand one's labour by a factor α if and only if one would have all others expand theirs by the same factor.

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Kantian behaviour is defined with respect to comparison of the present with a certain class of counterfactual alternatives.

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Kantian behaviour here does not ask an individual to be empathetic (taking on the preferences of other people): rather, it enjoins the individual to behave in the way that would maximize *her own welfare*, were all others to behave in a similar fashion.

Kantian equilibria and efficiency

Definition 1: A game $G = (S, V^1, ..., V^n)$ is *monotone increasing* (resp., *decreasing*) if

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Definition 2: A strategy profile $\mathbf{L} = (L^1, ..., L^n) \in S^n$ is *G-efficient* if there exists no other $\mathbf{L}' \in S^n$ that Pareto dominates $\mathbf{L}$ in G.

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Theorem 1: Suppose that $G = (S, V^1, ..., V^n)$ is monotone increasing or monotone decreasing. Let $\mathbf{L}^*$ be a Kantian equilibrium of G with $L^i > 0, \forall i = 1, ..., n$. Then $\mathbf{L}^*$ is G-efficient.

Kantian Equilibrium: existence

Theorem 2: Let $(V^1, ..., V^n)$ be concave real-valued pay-off functions defined on $\mathbb{R}^n_+$. For any $\mathbf{L} \in \mathbb{R}^n_{++}$, define $\alpha_i(\mathbf{L}) = \{a | a = \arg\max_{\alpha \in \mathbb{R}_+} V^i(\alpha \mathbf{L})\}$.

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Suppose:

(**) There exists $\mathbf{b} \in \mathbb{R}^n_{++}$ and $\mathbf{B} \in \mathbb{R}^n_{++}$ such that

$$(\mathbf{b} \leqq \mathbf{L} \leqq \mathbf{B} \Rightarrow (\forall i = 1, ..., n)(b^i \leqq \alpha_i(L)L^i \leqq B^i)).$$

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Then there exists a Kantian equilibrium for the game $G = (\mathbb{R}_+, V^1, ..., V^n)$ with $L^i > 0, \forall i = 1, ..., n$.

The game

Let $i = 1, 2$ be the set of players. The pure strategies available to them are $\{$cooperate, defect$\}$ and by allowing players to randomise we have $S = [0, 1]$, where $p \in S$ means that the player "cooperates" with probability p.

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For any strategy pair $(p, q) \in S^2$, the expected utilities of row (1) and column (2) players are given by:

$$
\begin{aligned}
V^1(p, q) &= pq + p(1-q)d + (1-p)qa, \\
V^2(p, q) &= pq + q(1-p)d + (1-q)pa.
\end{aligned}
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or, equivalently,

$$\arg\max_{\alpha \in [0,1]} \{\alpha^2(1 - a - d) + \alpha(a + d)\} = 1$$

Relational exploitation and ...

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(A) $(1,1)$ is a Kantian equilibrium of the PD game if and only if $(a+d) \leq 2$, and in this case, there is no other (non-trivial) Kantian equilibrium.

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(B) If $a + d > 2$, then the unique (non-trivial) Kantian equilibrium is given by

$$p^* = q^* = \frac{a + d}{2(a + d - 1)} < 1.$$

In particular, $p^* > \frac{1}{2}$.

Relational exploitation and ...

- Kantian reasoning promotes full cooperation provided the average of the utility of "cheating" (playing D when the opponent plays C) and the utility of being a "sucker" (playing C when the opponent plays D), i.e. $\frac{(a+d)}{2}$, is not too high (larger than the utility from full cooperation).

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- Even when full cooperation is not achieved (part (B)), players play cooperatively with a probability of at least one-half.

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Wlog, define $v^i(x^i, L^i) = u^i(x^i, \frac{l^i}{s^i})$, and then e is identical to the economy $e' = (v^1, ..., v^n, f)$.

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An *allocation rule* is a correspondence $\Theta : E \to\to \mathbb{R}_+^{2n}$ that associates to each economy a non-empty subset of feasible allocations.
A *game form* is a mapping G that associates to any vector of labour efforts $\mathbf{L} = (L^1, ..., L^n)$ in any economy e, an allocation in $F(e)$ of the form $\{(x^i, L^i)\}$. We denote $(x^i, L^i) = G^i(\mathbf{L}; e)$.

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Associated with a game form G is a set of pay-off functions $V^i : \mathbb{R}_+^n \times E \to \mathbb{R}_+$, $i = 1, ..., n$. Given an economy e, a game form G, and a vector $\mathbf{L} = (L^1, ..., L^n)$ then $V^i(\mathbf{L}; e) = u^i(G^i(\mathbf{L}; e))$.

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Associated with a game form G is a set of pay-off functions $V^i : \mathbb{R}_+^n \times E \rightarrow \mathbb{R}_+$, $i = 1, ..., n$. Given an economy e, a game form G, and a vector $\mathbf{L} = (L^1, ..., L^n)$ then $V^i(\mathbf{L}; e) = u^i(G^i(\mathbf{L}; e))$.
Hence, a game form evaluated at a particular economy induces a game with pay-off functions $\{V^i(\cdot; e)\}$.

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Denote by $K^+(G, e)$ the strictly positive Kantian equilibria of the game form G on e.

The proportional solution

Definition PS (Roemer and Silvestre 1993): For a given economy e, a *proportional solution* (PS) is an allocation $\{(x^i, L^i)\}$ such that:

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Define the game form G_{PS} by $G^i_{PS}(\mathbf{L}; e) = \left(\frac{L^i}{\sum_{j=1}^n L^j} f(\sum_{j=1}^n L^j), L^i \right)$ with $V^i_{PS}(\mathbf{L}; e) = u^i \left(\frac{L^i}{\sum_{j=1}^n L^j} f(\sum_{j=1}^n L^j), L^i \right)$.

Kantian implementation and efficiency

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Theorem 3. Let be a Pareto-efficient allocation rule defined on E. Let G be a non-wasteful game form that implements Θ with $\mathbf{L} > 0$ in Kantian equilibrium on E. Then $\Theta = \Theta^{PS}$.

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Thus, the only Pareto-efficient allocation rule that can be Kantian-implemented on this domain of economies is the proportional solution.

The public good problem

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Thus, the set of feasible allocations is $\{(\mathbf{L}, y) | C(y) \leq \sum_{i=1}^{n} L^i\}$.

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Suppose the strategy space consists of labour contributions. The *cost-share allocation rule* can be defined as $\Theta^{CS}(e) = \{(\mathbf{L}, y) | (\exists (b^1, ..., b^n) \in \mathbb{R}^n_+)((\mathbf{L}, y)$ is a linear cost-share equilibrium for e at $(b^1, ..., b^n))\}$.

Kantian implementation

Define the game form G^* on Γ as $G^{*i}(\mathbf{L}; e) = \left(C^{-1}\left(\sum_{j=1}^{n} L^j \right), L^i \right)$.

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Theorem 4. For $e \in \Gamma$: $K^+(G^*, e) = \Theta^{CS}(e)$.

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2. It unifies several equilibrium concepts in the literature concerning public goods and public bads.
3. Kantian equilibria are Pareto-efficient among the feasible allocations that can be achieved in a given game. We can view this as a result of the fact that the Kantian thought-experiment forces individuals to internalize externalities.

The road ahead

Here the focus is on multiplicative perturbation to the strategy profile: explore more general perturbations (Roemer 2012).

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How do Kantian equilibria emerge? Perhaps need a dynamic theory explaining Kantian equilibrium as the stationary state of a learning or (Kantian) optimisation process.